Seat No.: _____   Enrolment No.: _____

GUJARAT TECHNOLOGICAL UNIVERSITY
DIPLOMA ENGINEERING – SEMESTER – II • EXAMINATION – SUMMER- 2016

Subject Code: 3300001   **Date: 07-06 -2016**

Subject Name: Basic Mathematics

Time: 02:30 PM TO 05:00 PM   **Total Marks: 70**

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Each question carry equal marks (14 marks)

Q.1   Fill in the blanks using appropriate choice from the given options.   **14**

1     $\log_4\left(\dfrac{1}{2}\right) = \ldots\ldots\ldots$

      (a) $-\dfrac{1}{2}$ (b) $\dfrac{1}{2}$ (c) 2 (d) -2

2     $\log m - \log n = \ldots\ldots\ldots$

      (a) $\log mn$ (b) $\log\dfrac{m}{n}$ (c) $\log\dfrac{n}{m}$ (d) $\log(m-n)$

3     $\log_5 625 + \log_5 \sqrt{5} = \ldots\ldots\ldots$

      (a) $\dfrac{2}{9}$ (b) $\dfrac{5}{2}$ (c) $\dfrac{2}{5}$ (d) $\dfrac{9}{2}$

4     $\begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix} = \ldots\ldots$

      (a) -23 (b) 23 (c) 7 (d) -7

5   If $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ then adjA=…………

      (a) $\begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$

6   If A= $\begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$ then 3A=……….

(a) $\begin{bmatrix} 3 & 12 \\ 3 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 \\ 9 & -6 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 4 \\ 9 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 12 \\ 9 & -6 \end{bmatrix}$

7

If $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ then AB=............

(a) $\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$ (b) $\begin{bmatrix} -2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \end{bmatrix}$

8 $\sin 135^0 = ...$

(a) $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{-1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $-\sqrt{2}$

9 $\sin^{-1} x + \cos^{-1} x = ...$

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{2}{\pi}$ (c) $\dfrac{-2}{\pi}$ (d) π

10 Period of $\sin 3x = ...$

(a) $\dfrac{2\pi}{3}$ (b) $\dfrac{\pi}{3}$ (c) π (d) 2π

11 $\sin 3A = ...$

(a) $3\sin A - 4\sin^3 A$ ((b) $4\sin^3 A - 3\sin A$

(c) $4\sin A - 3\sin^4 A$ (d) $3\sin^4 A - 4\sin A$ m^2

12 Volume of sphere having radius r =...........

(a) $\dfrac{3}{4}\pi r^3$ (b) $\dfrac{\pi}{3} r^3$ (c) $\dfrac{4}{3}\pi r^3$ (d) $4\pi r^2$

13 Diameter of a circle is 28 cm. Area of a circle is.........

(a) $196\pi\ cm^2$ (b) $196\ cm^2$ (c) $28\pi\ cm^2$ (d) $28\ cm^2$

14 Area of a square having perimeter 2m is ....cm^2

(a) 1000 (b)2000 (c) 2500 (d) 5000

Q.2 **(a)** Attempt any two   **06**

1. Prove that $\log_b a . \log_c b . \log_a c = 1$

2. Find the circumference of a circle having area 38.5 cm^2

3. The surface area of the sphere is 616 m^2. Find the diameter of the sphere

(b) Attempt any two   **08**

1. If $\log\left(\dfrac{x+y}{2}\right)=\dfrac{1}{2}(\log x+\log y)$ then prove that $x=y$

2. Solve : $\log x+\log(x-5)=\log 6$

3. Length of one side of a rectangular plot is 35 m and length of its one diagonal is 37 m. Find the area of a plot.

Q.3 **(a)** Attempt any two   **06**

1. Expand $\begin{vmatrix} 5 & 3 & -1 \\ 4 & -3 & 0 \\ 6 & 1 & 2 \end{vmatrix}$ using Sarus's Method

2. If A=$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \end{bmatrix}$, B=$\begin{bmatrix} 3 & -2 & 4 \\ 1 & 5 & 0 \end{bmatrix}$ and $X+A+B=0$ find X

3. If $A=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$, find AB

(b) Attempt any two   **08**

1. If $A=\begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}$, $B=\begin{bmatrix} -1 & 5 \\ 4 & -3 \end{bmatrix}$ then prove that $\begin{pmatrix} 1 & 3 & -4 \end{pmatrix}\ (AB)^T=B^T A^T$

2. If A=$\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ then find A^{-1}

3. Solve $2x+3y=1$, $y-4x=2$ using matrix method

Q.4 **(a)** Attempt any two   **06**

1. Find $\sin 15^0$

2. Prove that $\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B$

3. Prove that $\dfrac{\sin(\pi+\theta)}{\sin(2\pi-\theta)}+\dfrac{\tan\left(\dfrac{\pi}{2}+\theta\right)}{\cot(\pi-\theta)}+\dfrac{\cos(2\pi+\theta)}{\sin\left(\dfrac{\pi}{2}+\theta\right)}=3$

(b) Attempt any two   **08**

1. Draw a graph of $y = \cos x$, $0 \le \cos x \le \pi$

2. Prove that $\dfrac{\sin 4A + 2\sin 5A + \sin 6A}{\cos 4A + 2\cos 5A + \cos 6A} = \tan 5A$

3. Prove that

   (i) $2\tan^{-1}\left(\dfrac{2}{3}\right) = \tan^{-1}\left(\dfrac{12}{5}\right)$

   (ii) $\sin\left(\tan^{-1} x + \cot^{-1} x\right) = 1$

Q.5 **(a)** Attempt any two   **06**

1. If $a = \begin{pmatrix} 3 & -1 & -4 \end{pmatrix}, b = \begin{pmatrix} -2 & 4 & -3 \end{pmatrix}$ and $c = \begin{pmatrix} -1 & 2 & -5 \end{pmatrix}$ find $|a + 2b - c|$

2. If $a = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix}, b = \begin{pmatrix} 2 & -1 & 1 \end{pmatrix}$ and $c = \begin{pmatrix} 1 & 1 & -2 \end{pmatrix}$, find $a.(b+c)$

3. Find X if $a = \begin{pmatrix} 2 & 3 & -1 \end{pmatrix}$ and $b = \begin{pmatrix} X & -1 & 3 \end{pmatrix}$ are perpendicular to each other

(b) Attempt any two   **08**

1. Prove that the angle subtended between $\begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$ and $\begin{pmatrix} 2 & -2 & 1 \end{pmatrix}$ is

   $\sin^{-1}\sqrt{\dfrac{26}{27}}$

2. Find a unit vector perpendicular to both $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$ and $\begin{pmatrix} -2 & 1 & -2 \end{pmatrix}$

3. The constant forces $\begin{pmatrix} 1 & -1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 & -3 \end{pmatrix}$ and $\begin{pmatrix} 4 & 5 & -6 \end{pmatrix}$ act on a particle. Under the action of these constant forces , particle moves from the point $\begin{pmatrix} 3 & -2 & 1 \end{pmatrix}$ to the point $\begin{pmatrix} 1 & 3 & -4 \end{pmatrix}$. Find the total work done by the forces.

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ગુજરાતી -3300001

પ્રશ્ન. ૧   યોગ્ય વિકલ્પ પસંદ કરી ખાલી જગ્યા પુરો.   14

(1) $\log_4\left(\dfrac{1}{2}\right) = \ldots\ldots\ldots$

(અ) $-\dfrac{1}{2}$ (બ) $\dfrac{1}{2}$ (ક) 2   (ડ) -2

(2) $\log m - \log n = \ldots\ldots\ldots$

(અ) $\log mn$ (બ) $\log \dfrac{m}{n}$ (ક) $\log \dfrac{n}{m}$ (ડ) $\log(m-n)$

(3) $\log_5 625 + \log_5 \sqrt{5} = \ldots\ldots\ldots$

(અ) $\dfrac{2}{9}$ (બ) $\dfrac{5}{2}$ (ક) $\dfrac{2}{5}$ (ડ) $\dfrac{9}{2}$

(4) $\begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix} = \ldots\ldots\ldots$

(અ) -23 (બ) 23 (ક) 7 (ડ) -7

(5) જો $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ તો adjA=$\ldots\ldots\ldots$

(અ) $\begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$ (બ) $\begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$ (ક) $\begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}$ (ડ) $\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$

(6) જો A=$\begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$ તો 3A=$\ldots\ldots\ldots$

(અ) $\begin{bmatrix} 3 & 12 \\ 3 & -2 \end{bmatrix}$ (બ) $\begin{bmatrix} 1 & 4 \\ 9 & -6 \end{bmatrix}$ (ક) $\begin{bmatrix} 3 & 4 \\ 9 & -2 \end{bmatrix}$ (ડ) $\begin{bmatrix} 3 & 12 \\ 9 & -6 \end{bmatrix}$

(7) જો $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ તો AB=$\ldots\ldots\ldots$

(અ) $\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$ (બ) $\begin{bmatrix} -2 \end{bmatrix}$ (ક) $\begin{bmatrix} 1 & 0 & -3 \end{bmatrix}$ (ડ) $\begin{bmatrix} 2 \end{bmatrix}$

(8) $\sin 135^0 = \ldots\ldots\ldots$

(અ) $\dfrac{1}{\sqrt{2}}$ (બ) $\dfrac{-1}{\sqrt{2}}$ (ક) $\sqrt{2}$ (ડ) $-\sqrt{2}$

(9) $\sin^{-1} x + \cos^{-1} x = \ldots\ldots\ldots$

(અ) $\dfrac{\pi}{2}$ (બ) $\dfrac{2}{\pi}$ (ક) $\dfrac{-2}{\pi}$ (ડ) π

(10) $\sin 3x$ નું આવર્તમાન $\ldots\ldots\ldots\ldots\ldots$

(અ) $\dfrac{2\pi}{3}$ (બ) $\dfrac{\pi}{3}$ (ક) π (ડ) 2π

(11) $\sin 3A = \ldots\ldots$

(અ) $3\sin A - 4\sin^3 A$ (બ) $4\sin^3 A - 3\sin A$ (ક) $4\sin A - 3\sin^4 A$

(ડ) $3\sin^4 A - 4\sin A$

(12) r ત્રિજ્યાવાળા ગોલક નું ઘનફળ $\ldots\ldots\ldots\ldots$

(અ) $\dfrac{3}{4}\pi r^3$ (બ) $\dfrac{\pi}{3} r^3$ (ક) $\dfrac{4}{3}\pi r^3$ (ડ) $4\pi r^2$

(13) વર્તુળ નો વ્યાસ 28 સેમી. છે. વર્તુળ નું ક્ષેત્રફળ = $\ldots\ldots$

(અ) $196\pi \ cm^2$ (બ) $196 \ cm^2$ (ક) $28\pi \ cm^2$ (ડ) $28 \ cm^2$

(14.) 2 m પરિમિતિવાળા ચોરસનું ક્ષેત્રફળ $\ldots\ldots$ cm^2

(અ)1000 (બ)2000 (ક) 2500 (ડ) 5000

પ્રશ્ન. ૨ અ કોઇપણ બે ગણો   6

(1) સાબિત કરો કે $\log_b a . \log_c b . \log_a c = 1$

(2) જેનું ક્ષેત્રફળ 38.5 cm^2 છે તે વર્તુળનો પરિઘ મેળવો.

(3) ગોલક ની વક્ર સપાટીનું ક્ષેત્રફળ $616\, m^2$ છે. ગોલક નો વ્યાસ

મેળવો.

બ કોઇપણ બે ગણો.   8

(1) જો $\log\left(\dfrac{x+y}{2}\right) = \dfrac{1}{2}(\log x + \log y)$ તો સાબિત કરો કે x=y

(2) સમીકરણ $\log x + \log(x-5) = \log 6$ ઉકેલો

(3) લંબચોરસ પ્લોટ ની એક બાજુ અને વિકર્ણ નું માપ અનુક્રમે 35 m

અને 37 m છે. પ્લોટ નું ક્ષેત્રફળ મેળવો.

પ્રશ્ન. 3 અ કોઇપણ બે ગણો   6

(1) $\begin{vmatrix} 5 & 3 & -1 \\ 4 & -3 & 0 \\ 6 & 1 & 2 \end{vmatrix}$ નું વિસ્તરણ સારસ પધ્ધતિથી કરો.

(2) જો $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 5 & 0 \end{bmatrix}$ અને $X + A + B = 0$ તો x મેળવો

(3) જો $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$, તો AB મેળવો

બ કોઇપણ બે ગણો   8

(1) જો $A = \begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 5 \\ 4 & -3 \end{bmatrix}$ તો સાબિત કરો કે $(AB)^T = B^T A^T$

(2) જો $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ તો A^{-1} મેળવો

(3) સમીકરણ $2x + 3y = 1$, $y - 4x = 2$ નો ઉકેલ ક્ષણિક પધ્ધતિથી મેળવો.

પ્રશ્ન. ૪ અ કોઇપણ બે ગણો   6

(1) $\sin 15^0$ મેળવો.

(2) સાબિત કરોકે $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

(3) સાબિત કરોકે $\dfrac{\sin(\pi+\theta)}{\sin(2\pi-\theta)} + \dfrac{\tan\left(\dfrac{\pi}{2}+\theta\right)}{\cot(\pi-\theta)} + \dfrac{\cos(2\pi+\theta)}{\sin\left(\dfrac{\pi}{2}+\theta\right)} = 3$

બ કોઇપણ બે ગણો   8

(1) $y = \cos x$, $0 \le \cos x \le \pi$ નો ગ્રાફ દોરો.

(2) સાબિત કરોકે $\dfrac{\sin 4A + 2\sin 5A + \sin 6A}{\cos 4A + 2\cos 5A + \cos 6A} = \tan 5A$

(3) સાબિત કરોકે

(i) $2\tan^{-1}\left(\dfrac{2}{3}\right) = \tan^{-1}\left(\dfrac{12}{5}\right)$

(ii) $\sin\left(\tan^{-1}x + \cot^{-1}x\right) = 1$

પ્રશ્ન. ૫ અ કોઇપણ બે ગણો   6

(1) જો $a = (3 \quad -1 \quad -4), b = (-2 \quad 4 \quad -3)$ અને $c = (-1 \quad 2 \quad -5)$ તો

$|a + 2b - c|$ મેળવો.

(2) જો $a = (1 \quad -1 \quad 1), b = (2 \quad -1 \quad 1)$ અને $c = (1 \quad 1 \quad -2)$, તો

$a.(b + c)$ મેળવો

(3) જો $a = (2 \quad 3 \quad -1)$ અને $b = (X \quad -1 \quad 3)$ પરસ્પર લંબ હોય તો

X મેળવો.

બ કોઇપણ બે ગણો   8

(1) સાબિત કરો કે સદિશો $(1 \quad 1 \quad -1)$ અને $(2 \quad -2 \quad 1)$ વચ્ચે અંતરાયેલો

ખુણો $\sin^{-1}\sqrt{\dfrac{26}{27}}$ છે.

(2) સદિશો $(1 \quad 2 \quad 3)$ અને $(-2 \quad 1 \quad -2)$ લંબ આવેલો એકમ સદિશ

મેળવો.

(3) એક કણ ઉપર અચળબળો $(1 \quad -1 \quad 1), (1 \quad 1 \quad -3)$ અને $(4 \quad 5 \quad -6)$

કાર્ય કરે છે. આ બળો ની અસર તે કણ બિંદું $(3 \quad -2 \quad 1)$ થી

ખસીને બિંદું $(1 \quad 3 \quad -4)(1 \quad 3 \quad -4)$ આગળ આવેછે. બળો દ્વારા

કુલ કાર્ય મેળવો.

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