Government Polytechnic, Ahmedabad
General Department

Mathematics (4300001): Semester I All Branches

Course Material

Faculty Name : **Term:**

Course Code : 4300001

Course Title : Mathematics **Lectures Per Week :** 03

Total Lectures Per Term : 42

SUGGESTED SPECIFICATION TABLE FOR QUESTION PAPER DESIGN

Unit No.	Unit Title	Teaching Hours	Distribution of Theory Marks			
			R Level	U Level	A Level	Total Marks
I	Determinant and Function	9	4	7	5	16
II	Trigonometry	12	4	5	5	14
III	Vectors	7	4	6	4	14
IV	Coordinate Geometry	8	4	5	5	14
V	Limit	6	3	4	5	12
Total		42	**19**	**27**	**24**	**70**

Legends: *R=Remember, U=Understand, A=Apply and above (Revised Bloom's taxonomy)*

Note*: This specification table provides general guidelines to assist students for their learning and to teachers to teach and question paper designers/setters to formulate test items/questions to assess the attainment of the UOs. The actual distribution of marks at different taxonomy levels (of R, U and A) in the question paper may slightly vary from above table.*

SUGGESTED LEARNING RESOURCES

S. No.	Title of Book	Author	Publication with place, year and ISBN
1	Engineering Mathematics (Third edition).	Croft, Anthony	Pearson Education, New Delhi, 2014. ISBN 978-81-317-2605-1
2	A Text Book of Vector Analysis	Narayan Shanti and Mittal P.K	S. Chand Publication, ISBN 978-8121922432
3	Calculus and Analytic Geometry	G. B. Thomas, R. L. Finney	Addison Wesley, 9th Edition, 1995. ISBN 978-8174906168
4	Understanding Engineering Mathematics	John Bird	Routledge; 1st edition ISBN 978-0415662840
5	Advanced Engineering Mathematics	Krezig, Ervin	Wiley Publ., New Delhi,2014, ISBN: 978-0-470-45836-5

SUGGESTED MICRO-PROJECTS

Only one micro-project is planned to be undertaken by a student that needs to be assigned to him/her in the beginning of the semester. In the first four semesters, the micro-project are group-based (group of 3 to 5). However, in the fifth and sixth semesters, the number of students in the group should *not exceed three.*

The micro-project could be industry application based, internet-based, workshop-based, laboratory-based or field-based. Each micro-project should encompass two or more COs which are in fact, an integration of PrOs, UOs and ADOs. Each student will have to maintain dated work diary consisting of individual contribution in the project work and give a seminar presentation of it before submission. The duration of the micro project should be about 14-16 *(fourteen to sixteen) student engagement hours* during the course. The students ought to submit micro-project by the end of the semester (so that they develop the industry- oriented COs).

A suggestive list of micro-projects is given here. This should relate highly with competency of the course and the COs. Similar micro-projects could be added by the concerned course teacher:

a) Draw graphs of given Functions like $2x-1, x^2, \sin x, \cos x$ etc and verify using suitable Open-source software like GeoGebra, DPLOT and GRAPH.
b) Prepare the Charts of formulae for limit, Vector, Trigonometry, Co-ordinate Geometry, and Logarithm.
c) Prepare the cardboard models based on Mathematical concepts.
d) Draw various lines, circles using GeoGebra software.
e) Prepare projects on height and distance using Trigonometry.

f) Use PHET website for simulation of Vector Algebra.
g) Prepare a presentation/seminar on any relevant topic of interdisciplinary nature.
h) Prepare a write up on the Historical path of Calculus.
i) Prepare models of graphical representation for the existence of limits of given functions.
j) Prepare charts showing formulas of multiple and sub multiple trigonometric functions and its usefulness.
k) Formulate models to describe mathematical relationships and analyze data.

Unit-1: Determinant and Function

Unit No.	Lect. No.	Topics
1.	L1 TO L9	**Unit I Determinant and Function** **Course Outcome (CO a):** Interpret the function graphically, numerically and analytically.
	1 TO 2	**Determinants**
	1	**Definition and order of determinants.** Illustrations of determinants **Expansion of determinants of order 2.** **$\Rightarrow$ 2×2 Determinant is of the type** $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ having value ad-bc. (a, b, c, d$\in$R) Minor and cofactor of an element of determinant Illustrations on above topics
	2	**$\Rightarrow$ 3×3 Determinant is of the type** $\therefore \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)$ Illustrations on above topics
	3 TO 5	**Function**
	3	*Definition of function, domain co-domain, range set* Illustrations on above topics
	4	Different types of functions. Illustrations on above topics
	5	More illustrations on above topics
	6 TO 9	**Logarithm**

	6 TO 9	**Reorientation of indices** **Properties of Indices:** i) $a^m.n^n = a^{m+n}$ v) $\frac{a^m}{a^n} = a^{m-n}$ ii) $(a^m)^n = a^{mn}$ vi) $a^0 = 1$ iii) $(ab)^n = a^n b^n$ iv) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ a, b $\in$ R and m, n $\in$ R. **Definition of logarithm relation with exponential form of number.** **Properties of logarithms.** i). $a^m = n \Leftrightarrow \log_a n = m$. ii). $\log_a x + \log_a y = \log_a xy$. iii). $\log_a x - \log_a y = \log_a \frac{x}{y}$ iv) $\log_a x^m = m\log_a x$ v) $\log_a x = \frac{\log_b x}{\log_b a}$ vi) $\log_a a = 1$. Illustrations on above topics

Unit 01- Determinant and Function

Course Outcome: CO a) Interpret the function graphically, numerically and analytically.

Determinant

Question Set for 01 Mark:

Evaluate the following:

1	Evaluate $\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}$	Ans.-2		**2**	$\begin{vmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{vmatrix}$	Ans.1
3	If $\begin{vmatrix} x & 1 \\ 4 & 2 \end{vmatrix} = 0$, find x.	Ans.2		**4**	$\begin{vmatrix} 1 & 7 & -3 \\ -4 & 6 & 2 \\ 2 & -5 & 3 \end{vmatrix}$	Ans.116
5	$\begin{vmatrix} 3 & 4 & 5 \\ 0 & 3 & 2 \\ 2 & -4 & -5 \end{vmatrix}$	Ans. -35		**6**	$\begin{vmatrix} 3 & 2 & 1 \\ -1 & 2 & 6 \\ 3 & 0 & 5 \end{vmatrix}$	Ans.70
7	$\begin{vmatrix} x-2 & 3 \\ 0 & x+2 \end{vmatrix} = 0$ find x.	Ans.±2		**8**	$\begin{vmatrix} 5 & x+3 \\ x+3 & 5 \end{vmatrix} = 0.$	Ans. 2 or - 8

Question Set for 03 Marks:

1	If $\begin{vmatrix} x-2 & 2 & 2 \\ -1 & x & -2 \\ 2 & 0 & 4 \end{vmatrix} = 0$, find x	Ans: $x = 0$ or $x = 3$
2	Find out minor and co factor of each element $\begin{vmatrix} 1 & -2 & 3 \\ 3 & 0 & -2 \\ 4 & 6 & 8 \end{vmatrix}$	

Function

- **01 Mark**

1) If f(*x*) = $\log\left(e^{x}\right)$ then f (0) =_____

(a) 0 (b) 1 (c) 2 (d) None of these

2) If $f(x) = \log\left(\frac{x}{x-1}\right)$ then $f(-x)$ = _____

(a) $\log\left(\frac{x}{x-1}\right)$ (b) $\log\left(\frac{x}{x+1}\right)$ (c) $\log\left(\frac{x-1}{x}\right)$ (d) $\log\left(\frac{x+1}{x}\right)$

Answers: - ***1)*** a ***2)*** b

- **03 Marks**

1) If $f(x) = \frac{1-x}{1+x}$ then prove that

(i) $f(x) + f\left(\frac{1}{x}\right) = 0$ (ii) $f(x) - f\left(\frac{1}{x}\right) = 2f(x)$

(iii) $f(x).f(-x) = 1$

2) If $f(x) = \log x$ then prove that (i) $f(x.y) = f(x) + f(y)$ (ii)$f\left(\frac{x}{y}\right) = f(x) - f(y)$

3) If $f(x) = \tan x$ then prove that (i) $f(x+y) = \frac{f(x)+f(y)}{1-f(x).f(y)}$ (ii)$f(2x) = \frac{2f(x)}{1-[f(x)]^2}$

4) If $f(x) = \frac{1}{1+x}$ then prove that $f(x) + f\left(\frac{1}{x}\right) = 1$

5) If $f(x) = \log\left(\frac{1-x}{1+x}\right)$ then prove that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$

6) If $f(x) = \log\left(\frac{x-1}{x}\right)$ then prove that $f(x) + f(-x) = f(x^2)$

7) If $f(x) = \sin x$ then prove that $2\,f(x).f\left(\frac{\pi}{2} + x\right) = f(2x)$

8) If $(x) = \log_2 x$,$g(x) = x^4$ then prove that $f\big(g(2)\big) = 4$.

9) If $f(x) = \frac{ax+b}{bx+a}$ then prove that $f(x).f\left(\frac{1}{x}\right) = 1$

Logarithms

Question Set for 01 Mark:

Evaluate the following Logarithmic expressions:

1	$\log_2 64$	Ans. 6		**2**	$\log_3(1/27)$	Ans. -3
3	$\log_{10}(0.001)$	Ans . -3		**4**	log32 - log16	Ans. log2
5	log32 ÷ log16	Ans. 5/4		**6**	$27^{(\log_3 2)}$	Ans. 8
7	$3^{(\log_9 4)}$	Ans. 2		**8**	$\log_7 49$	Ans. 2
9	$\log_2 (1/8)$	Ans. -3		**10**	$\log_8 2$	Ans. 1/3
11	log2+log(1/2)	Ans. 0		**12**	$\log_5 625$	Ans. 4
13	$\log_{\frac{1}{3}} 9$	Ans. -2		**14**	If $\log_2 x=1$, then x=?	Ans. 2
15	log5+log3	Ans. log15		**16**	$\log_3(\log_3 27)$	Ans. 1
17	If $\sqrt{\log_2 x} = 3$, then x = ...	Ans. x =512		**18**	$16^{-\log_{16}\frac{2}{5}}$	Ans. $\frac{5}{2}$

Question Set for 03 Marks:

1	*Find the value of log(9/14) - log(15/16) + log(35/24)*	Ans. 0
2	*Prove that log(75/16) – 2log(5/9) + log(32/243) = log2*	
3	*Simplify :* $log\left(\frac{450}{32}\right) + log\left(\frac{25}{128}\right) + log\left(\frac{64}{225}\right) + log\left(\frac{32}{25}\right)$	Ans. 0
4	Prove that $\log[x + \sqrt{x^2-1}\ ] + \log[\ x - \sqrt{x^2-1}\ ] = 0$	
5	Prove that $\log\frac{a^2}{bc} + \log\frac{b^2}{ca} + \log\frac{c^2}{ab} = 0$	
6	Simplify $\log_3 84 - \log_3 28 - 3^{\log_3 1}$	Ans. 0
7	Solve : $2\log 3 \times \log x = \log 27 \times \log 9$	Ans. 27
8	Solve: $\log_2(\log_3(\log_2 x)) = 1$	*Ans. 512*
9	Prove that $\frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24} = 2$.	
10	Prove that: $\log_y x^2 \times \log_z y^3 \times \log_x z^4 = 24$	

11	Prove that $\frac{1}{\log_2 6}+\frac{1}{\log_3 6}=1$	
12	Prove that: $\log_x y \times \log_y z \times \log_z w = \log_x w$	
13	If $\log_a x^3 - \log_a 25 = \log_a x$ then find the value of x.	*Ans. 5*
14	If $\frac{4log3\times logx}{log9} = \log 27$, Then find the value of x.	*Ans.* $3\sqrt{3}$
15	If $\log_2 x = 2$ and $\log_x y = 2$, then find the value of y.	*Ans. 16*
16	If $\log_{10} 2 = 0.30103$ and $\log_{10} 3 = 0.47712$ then evaluate $\log_{10} 5$ and $\log_{10} 1.2$ without using logtable.	
17	If $\log_{2x} x = a, \log_{3x} 2x = b,$ and $\log_{4x} 3x = c$ then prove that $abc+1=2bc$	

Question Set for 04 Marks:

1	Prove that $\frac{1}{\log_a abc}+\frac{1}{\log_b abc}+\frac{1}{\log_c abc}=1.$	
2	Prove that $\frac{1}{\log_{bc} p}+\frac{1}{\log_{ca} p}+\frac{1}{\log_{ab} p}=2\log_p abc.$	
3	If $\log\left(\frac{a+b}{2}\right)=\frac{1}{2}(\log a+\log b)$, prove that $a = b$.	
4	Prove that : $\log_a p + \log_{a^2} p^2 + \log_{a^3} p^3 + \log_{a^4} p^4 = 4$	
5	Prove that $\log_x\left(\frac{a-b}{b-c}\right)+\log_x\left(\frac{b-c}{c-a}\right)+\log_x\left(\frac{c-a}{a-b}\right)=0.$	
6	Prove that $\log_{\sqrt{q}} p^2 \cdot \log_{\sqrt{r}} q^2 \cdot \log_{\sqrt{p}} r^2 = 64$	
7	Prove that $\frac{1}{\log_x yz+1}+\frac{1}{\log_y zx+1}+\frac{1}{\log_z xy+1}=1$	
8	Solve the following equation: $\log(2x+1)+\log(3x-1)=0.$	*Ans.* $x=\frac{1}{2}$
9	If $\log\left(\frac{x+y}{3}\right)=\frac{1}{2}(\log x+\log y)$, then prove that $x^2+y^2=7xy$	
10	If $\frac{2\log_5 x+\log_5 9}{\log_5 3x} = \log_5 x$ then find the value of x.	*Ans. x=25*
11	Prove that: $\log_2\left(\frac{1}{320}\right)+\frac{1}{\log_{10} 2}+5=0$	