Unit – 2: Trigonometry

2.	**L10 TO L21**	**Trigonometry** **Course Outcome (CO b):** Demonstrate the ability to algebraically analyze basic functions used in Trigonometry.
	10	**Measurement of angles** ❖ **<u>Units of Measuring Angles:</u>** ➢ Degrees (θ^0) ➢ Radians (θ^1 or θ^c) Relation Between Degree and Radian: $2\pi^1 = 360^0 \Leftrightarrow \pi^1 = 180^0 \Leftrightarrow 1^1 = \frac{180^0}{\pi}$ And $1^0 = \frac{\pi^1}{180}$ Illustrations on relation between degree & radian

	12	**Trigonometric functions** **Value of t –ratio for standard measurement of angles**

θ / *fun.* $\rightarrow$	**sin**	**cos**	**tan**	**cot**	**sec**	**cosec**
0	0	1	0	∞	1	∞
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
$\frac{\pi}{2}$	1	0	∞	0	∞	1
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	-2	$\frac{2}{\sqrt{3}}$
$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	2
π	0	-1	0	∞	-1	∞

Illustrations related to values of trigonometric functions

	13	**Graph of trigonometric functions** **Period, Principal period, Range and zeros of trigonometric functions** Illustrations of sketch graph of T – functions

	14 TO 15	**Addition formulae and Negative angle formulae** Trigonometric identity: ▪ $\sin\theta\,\mathrm{cosec}\,\theta = \cos\theta\sec\theta = \tan\theta\cot\theta = 1$ ▪ $\sin^2(x) + \cos^2(x) = 1$ ▪ $\sec^2(x) - \tan^2(x) = 1$ ▪ $\mathrm{cosec}^2(x) - \cot^2(x) = 1$ Addition formulae: ▪ $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$ ▪ $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$ ▪ $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ ▪ $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ ▪ $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$ ▪ $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$ Negative angle formulae Illustrations on above topics
	16	Allied angle formulae: ▪ $\sin(\frac{\pi}{2}+\theta) = \cos\theta$ $\cos(\frac{\pi}{2}+\theta) = -\sin\theta$ ▪ $\sin(\pi+\theta) = -\cos\theta$ $\cos(\pi+\theta) = -\sin\theta$ ▪ $\sin(\pi-\theta) = \sin\theta$ $\cos(\pi-\theta) = -\cos\theta$ ▪ $\tan(\frac{\pi}{2}+\theta) = -\cot\theta$ $\tan(\pi+\theta) = \tan\theta$ ▪ $\tan(\pi-\theta) = -\tan\theta$ ▪ $\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos(\theta)$ Illustrations on above formulae

	17	**Factor formulae** ➢ **De-factorization formulas:** ▪ $2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$ i.e {2SC = S + S} ▪ $2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$ i.e {2CS = S - S} ▪ $2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$ i.e {2CC = C + C} ▪ $2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$ i.e {2SS = C - C} ➢ **Factorization formulas:** ▪ $\text{SinA} + \text{sinB} = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ ▪ $\text{SinA} - \text{sinB} = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ ▪ $\text{cosA} + \text{cosB} = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ ▪ $\text{cosA} - \text{cosB} = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$ OR $\text{cosA} - \text{cosB} = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ Illustrations on above formulae

	18 TO 19	**Formulae for multiple and submultiples angles** ▪ $\sin 2\theta = 2sin\theta\cos\theta$ Similarly, $\sin\theta = 2sin\frac{\theta}{2}\cos\frac{\theta}{2}$ $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ $\sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$ ▪ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ Similarly, $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$ $\cos 2\theta = 1 - 2\sin^2\theta$ $\cos\theta = 1 - \sin^2\frac{\theta}{2}$ $\cos 2\theta = 2\cos^2\theta - 1$ $\cos\theta = 2\cos^2\frac{\theta}{2} - 1$ $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ $\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$ ▪ $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ ▪ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ Similarly, $\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$ ▪ $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ ▪ $\tan 3\theta = \frac{3\tan\theta - \tan^2\theta}{1-3\tan^2\theta}$ Illustrations on above formulae

	20 TO 21	**Inverse trigonometric functions** If x>0, y>0 then ❑ $\sin^{-1}(-x) = -\sin^{-1}x, \lvert x \rvert \le 1$ $\qquad$ $\cos^{-1}(-x) = \pi - \cos^{-1}x, \lvert x \rvert \le 1$ ❑ $\tan^{-1}(-x) = -\tan^{-1}x, x \in R$ $\qquad$ $\cot^{-1}(-x) = \pi - \cot^{-1}x, x \in R$ ❑ $\operatorname{cosec}^{-1}x = \sin^{-1}\frac{1}{x}, \lvert x \rvert \ge 1$ $\qquad$ $\sec^{-1}x = \cos^{-1}\frac{1}{x}, \lvert x \rvert \ge 1$ ❑ $(i)\cot^{-1}x = \tan^{-1}\frac{1}{x}, x > 0$ $\qquad$ and $(ii)\cot^{-1}x = \tan^{-1}\frac{1}{x} + \pi, x < 0$ ❑ $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, \lvert x \rvert \le 1$ $\qquad$ $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}, \lvert x \rvert \ge 1$ ❑ $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}, x \in R$ ❑ $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$, If xy<1 ❑ $\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\frac{x+y}{1-xy}$, If xy>1 ❑ $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{2}$, If xy = 1 $\qquad$ $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$ ❖ **<u>Inter relation Formulas:</u>** ❑ $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}, 0 < x < 1$ ❑ $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x}, 0 < x < 1$ ❑ $\tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}} = \sin^{-1}\frac{x}{\sqrt{1+x^2}}, x > 1$ Illustrations on above formulae:

Unit 02: Trigonometry

Course Outcome: **CO b)** Demonstrate the ability to algebraically analyze basic functions used in Trigonometry.

Question Set for 01 Mark

1	Convert into degree:- $\frac{3\pi}{4}, \frac{7\pi}{6}$	Ans. $135^0, 210^0$
2	Convert into radian:- 150° , 20°	Ans. $\frac{5\pi}{6}, \frac{\pi}{9}$
3	If $\cos\theta = \frac{\sqrt{3}}{2}$, $\sin\theta = -\frac{1}{2}$ then θ lies in which quadrant	Ans. 4^{th} Quadrant
4	$\text{Cos } 90^0 \times \cos 60^0 \times \sin 30^0$	Ans. 0
5	$\tan 225°$	Ans . 1
6	$\cot(-30°)$	Ans $-\sqrt{3}$
7	$sin^2 57° + sin^2 33°$	Ans. 1
8	$\cos^2 30^0 + \cos^2 60^0$	Ans. 1
9	If A+B+C = π , then $\cos\frac{B+C}{2}$ is	Ans. sin(A/2)
10	$\cos x + \cos(\pi - x) + \cos(2\pi - x) + \cos(3\pi - x)$	Ans. 0
11	Period of sin2x	Ans π
12	Period of $\tan\left(3x + \frac{\pi}{6}\right)$	Ans. $\frac{\pi}{3}$
13	$\sin^{-1}\frac{1}{2}$	Ans. $\frac{\pi}{6}$
14	$\cos^{-1}\left(-\frac{1}{2}\right)$	Ans. $\frac{2\pi}{3}$
15	$\cos\left(\cos^{-1}\frac{2}{3}\right)$	Ans $\frac{2}{3}$
16	$\sin\left(\tan^{-1} p + \cot^{-1} p\right)$	Ans. 1

Question Set for **<u>03 Marks</u>**

1	Prove that $\tan 225^0 \times \cot 405^0 + \tan 765^0 \times \cot 675^0 = 0$
2	Prove that $\cos\frac{19\pi}{6}\cdot\sin\frac{17\pi}{6} - \sin\frac{11\pi}{6}\cdot\cos\frac{13\pi}{6} = 0$
3	Prove that $\sin^2\frac{\pi}{4} + \sin^2\frac{3\pi}{4} + \sin^2\frac{5\pi}{4} + \sin^2\frac{7\pi}{4} = 2$
4	Prove that $\tan\frac{\pi}{20}\ \tan\frac{3\pi}{20}\ \tan\frac{5\pi}{20}\ \tan\frac{7\pi}{20}\ \tan\frac{9\pi}{20} = 1$
5	Prove that $\sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$..
6	Prove that $\cos A\cdot\sin(B-C) + \cos B\cdot\sin(C-A) + \cos C\cdot\sin(A-B) = 0$
7	Prove that $\tan 55^\circ = \frac{\cos 10^\circ + \sin 10^\circ}{\cos 10^\circ - \sin 10^\circ}$
8	Prove that $\frac{\sin 79^\circ + \sin 19^\circ}{\cos 19^\circ - \cos 79^\circ} = \sqrt{3}$
9	Prove that $(1+\tan 25^\circ)(1+\tan 20^\circ) = 2$.
10	If $\tan x = \frac{5}{6}$ and $\tan y = \frac{1}{11}$, then prove that $x + y = \frac{\pi}{4}$
11	Prove that $\sin 3A = 3\sin A - 4\sin^3 A$
12	Prove that $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$
13	Prove that $\frac{\sin 19^\circ + \cos 11^\circ}{\cos 19^\circ - \sin 11^\circ} = \sqrt{3}$
14	Prove that $\sin 4\theta = \frac{4\tan\theta(1-\tan^2\theta)}{(1+\tan^2\theta)^2}$
15	Prove that $\frac{1+\sin 2A - \cos 2A}{1+\sin 2A + \cos 2A} = \tan A$
16	Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$.

Question Set for **04 Marks**

1	Prove that $\frac{\sin\left(\theta-\frac{\pi}{2}\right)}{\cos(\theta-\pi)}+\frac{\tan\left(\frac{\pi}{2}-\theta\right)}{\cot(\pi-\theta)}+\frac{\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)}{\sec(\pi+\theta)}=-1$
2	Prove that $\frac{\sin\theta+\sin 2\theta+\sin 4\theta+\sin 5\theta}{\cos\theta+\cos 2\theta+\cos 4\theta+\cos 5\theta}=\tan 3\theta$.
3	Prove that $\sin(180^0-\theta)\cos(-\theta)\cot(180^0-\theta)+\cos(360^0+\theta)\operatorname{cosec}(180^0-\theta)\cot(90^0-\theta)=\sin^2\theta$
4	Prove that $\frac{\tan(\pi-\theta)}{\tan(\pi+\theta)}\times\frac{\cot(\pi+\theta)}{\cot(\pi-\theta)}\times\frac{\tan(2\pi+\theta)}{\cot(2\pi-\theta)}=-\tan^2\theta$
5	If A, B, C & D are angles of cyclic quadrilateral then prove that $\cos A+\cos B+\cos C+\cos D=0$
6	For any ΔABC Prove that $\tan A+\tan B+\tan C=\tan A\cdot\tan B\cdot\tan C$
7	Prove that $4\sin 2A\sin(60^0+2A)\sin(60^0-2A)=\sin 6A$
8	If $\tan\theta=\frac{1}{2}$ then Prove that $7\cos 2\theta+8\sin 2\theta=\frac{53}{5}$
9	Prove that (a) $\cos 20^0+\cos 60^0+\cos 100^0+\cos 140^0=\frac{1}{2}$
10	Prove that $\sin 10^0\sin 30^0\sin 50^0\sin 70^0=\frac{1}{16}$
11	Prove that $\tan^{-1}\left(\frac{3}{4}\right)+\sin^{-1}\left(\frac{4}{5}\right)=\frac{\pi}{2}$
12	$\sin^{-1}\left(\frac{12}{13}\right)+\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{63}{13}\right)=\pi$
13	Draw the graph of $y=\sin x$, $0\le x\le 2\pi$
14	Draw the graph of $y=\cos x$, $0\le x\le 2\pi$
15	Draw the graph of $y=\sin 2x$, $-\pi\le x\le\pi$
16	Draw the graph of $y=3\cos(x/2)$, $0\le x\le 2\pi$