Unit-3: Vectors

3.	**L22 TO L28**	**Vectors** **Course Outcome (CO c):** Demonstrate the ability to Crack engineering related problems based on concepts of Vectors.
	22	**Vector and scalar quantities** **<u>Scalar:</u>** Any quantity which is represented by only magnitude is called a scalar e.g. time, length, distance, …. **<u>Vector:</u>** Any quantity which has magnitude as well as direction is called a vector e.g. Force, velocity, acceleration, …. Vectors are generally denoted by $\bar{x}, \bar{y}, \bar{z}$ etc. Where $\bar{x} = (x_1, x_2, x_3)$, $\bar{y} = (y_1, y_2, y_3)$, and $\bar{z} = (z_1, z_2, z_3)$ Where $x_{i,} y_i$ & $z_i \in R$ Illustration of Scalar and vector quantity. Physical, Geometrical and Mathematical representation of vector, Position vectors in terms of i, j & k Illustration of above definitions
	23	**Magnitude and direction of vectors** **<u>Magnitude of vector</u>**: If $\bar{x} = (x_1, x_2, x_3)$ then $\lvert\bar{x}\rvert = \sqrt{x_1^2 + x_2^2 + x_3^2}$ where $\lvert\bar{x}\rvert$ is magnitude of vector $\bar{x}$. **<u>Types of vector:</u>** Null, Unit, Opposite, Parallel, Orthogonal vectors - Units vectors I, j & k **<u>Unit vector</u>**: If $\lvert\bar{x}\rvert = 1$ then vector $\bar{x}$ is called unit vector and it is denoted by x, By definition x = $\frac{\bar{x}}{\lvert\bar{x}\rvert}$. Unit vectors in direction of X, Y & Z axes is denoted by i, j & k and is defined as i = (1, 0, 0), j = (0,1, 0) and k = (0, 0, 1). **<u>Algebraic operations of vectors:</u>** **<u>Operations between vectors:</u>** (i) **Equality:** Two vectors $\bar{x}$ & $\bar{y}$ are equal i.e. $\bar{x} = \bar{y}$ if $x_1 = y_1, x_2 = y_2$ & $x_3 = y_3$ (ii) **Addition:** $\bar{x} + \bar{y} = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$ (iii) **Multiplication by scalar:** $\alpha\bar{x} = (\alpha x_1, \alpha x_2, \alpha x_3)$

	24	**<u>Vector addition satisfies following properties:</u>** (i.) **Closure Property:** If $\overline{x}$ & $\overline{y}$ are vectors then $\overline{x}+\overline{y}$ is also a vector i.e. $\overline{x}=(x_1,x_2,x_3)$ & $\overline{y}=(y_1,y_2,y_3)$, are vector then $\overline{x}+\overline{y}=(x_1+y_1,x_2+y_2,x_3+y_3)$ is also a vector. (ii.) **Commutative Property:** If $\overline{x}$ & $\overline{y}$ are vector then $\overline{x}+\overline{y}=\overline{y}+\overline{x}$ (iii.) **Associative Property:** If $\overline{x},\overline{y}$ & $\overline{z}$ are vectors then $(\overline{x}+\overline{y})+\overline{z}=\overline{x}+(\overline{y}+\overline{z})$. (iv.) **Additive identity:** $\theta=(0,0,0)$ is null vector so, $\overline{x}+\theta=\theta$. **Additive inverse:** $\overline{x}+(-\overline{x})=\theta$. Examples on above topics.
	25	**Dot & cross product of two vectors** **<u>Angle between two vectors</u>:** If θ is an angle between two vectors then, $\theta = \cos^{-1}\left(\frac{\overline{x}\cdot\overline{y}}{\lvert\overline{x}\rvert\lvert\overline{y}\rvert}\right)$ **<u>Inner Product (Dot product)</u>**: If $\overline{x}=(x_1,x_2,x_3)$ and $\overline{y}=(y_1,y_2,y_3)$, then dot product (inner product) of $\overline{x}$ & $\overline{y}$ is defined as $\overline{x}\cdot\overline{y}=x_1y_1+x_2y_2+x_3y_3$ $=\lvert\overline{x}\rvert\lvert\overline{y}\rvert\cos\theta$, θ is an angle between vectors $\overline{x}$ & $\overline{y}$. **<u>Properties of dot product</u>**: (i) $\overline{x}\cdot\overline{y}=\overline{y}\cdot\overline{x}$, (ii) $(\overline{x}+\overline{y})\cdot\overline{z}=\overline{x}\cdot\overline{z}+\overline{y}\cdot\overline{z}$, (iii) $\overline{x}\neq\theta$ then $\overline{x}\cdot\overline{x}>0$ and $\overline{x}\cdot\overline{x}=\lvert\overline{x}\rvert=x_1^2+x_2^2+x_3^2>0$. **<u>Outer Product (vector product or cross product)</u>**: If $\overline{x}=(x_1,x_2,x_3)$ & $\overline{y}=(y_1,y_2,y_3)$, are two vectors Then outer product of this vectors is denoted by $\overline{x}\times\overline{y}$ and is defined as $\overline{x}\times\overline{y}=\begin{vmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$ **<u>Or</u>** $\overline{x}\times\overline{y}=(x_2y_3-x_3y_2,x_3y_1-x_1y_3,x_1y_2-x_2y_1)$ **<u>Direction cosines of vector</u>**

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<u>Properties of outer product:</u>
If $\bar{x}, \bar{y} \;\&\; \bar{z} \in R^3$ are three vectors then
(i) $\bar{x} \times \theta = \theta$, (ii) $\bar{x} \times \bar{x} = \theta$, (iii) $\bar{x} \times \bar{y} = -\bar{y} \times \bar{x}$,
(iv) $\bar{x} \times \alpha \bar{y} = \alpha(\bar{x} \times \bar{y})$, (v) $\bar{x} \times (\bar{y} + \bar{z}) = (\bar{x} \times \bar{y}) + (\bar{x} \times \bar{z})$.

<u>Lagrange’s identity</u>: If $x_1, x_2, x_3, y_1, y_2, y_3$ are real numbers then
$(x_1^2 + x_2^2 + x_3^2)(y_1^2 + y_2^2 + y_3^2) - (x_1y_1 + x_2y_2 + x_3y_3)^2 = (x_2y_3 - x_3y_2)^2 + (x_3y_1 - x_1y_3)^2 + (x_1$
or
$|\bar{x}|^2 |\bar{y}|^2 - (\bar{x} \cdot \bar{y})^2 = |\bar{x} \times \bar{y}|^2$
<u>Box Product of three vectors</u>: Box product of three vectors $\bar{x}, \bar{y} \;\&\; \bar{z} \in R^3$ is $\bar{x} \cdot (\bar{y} \times \bar{z})$ and it is denoted by symbol $\left[\bar{x}\bar{y}\bar{z}\right]$.
<u>Properties of box product</u>: (i) $\left[\bar{x}\bar{y}\bar{z}\right] = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$, (ii) $\left[\bar{x}\bar{y}\bar{z}\right] = \left[\bar{y}\bar{z}\bar{x}\right] = \left[\bar{z}\bar{x}\bar{y}\right]$,
(iii) $\bar{x} \times (\bar{y} \times \bar{z}) = (\bar{x} \cdot \bar{z})\bar{y} - (\bar{x} \cdot \bar{y})\bar{z}$
Examples on above topics
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- Definition of work done by force and moment of force
<u>Application of Vectors</u>
(i) <u>Magnitude and direction of resultant force</u>: If force $\bar{F_1}, \bar{F_2}, \bar{F_3}, \bar{F_4}, ...\bar{F_n}$ act on a particle then the resultant force acting on the particle is
$\bar{F} = \bar{F_1} + \bar{F_2} + \bar{F_3} + \bar{F_4} + ... + \bar{F_n}$. Here $|\bar{F}|$ is magnitude of resultant force and direction of resultant force is direction of vector $\bar{F}$.
(ii) <u>Work</u>: If a particle gets displacement($\bar{d}$) from point A to B under the force $\bar{F}$ then, work W done by force $\bar{F}$ is
$W = \bar{F} \cdot \bar{d} = |\bar{F}||\bar{d}| \cos \theta$, where θ is an angle between vectors $\bar{F}$ & $\bar{d}$.
(iii) <u>Moment of force</u>: Moment of force $M = \bar{d} \times \bar{F} = |\bar{F}||\bar{d}| \sin\theta \cdot n$,
n is unit vector in direction of M.
Examples on above topics.
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Unit 03: Vectors

Course Outcome: CO c) Demonstrate the ability to Crack engineering related problems based on concepts of Vectors.

Question Set for **01 Mark**

1	If $\bar{a} = 2i + 3j + k$, $\bar{b} = 2i - 3j + 2k$, then $\bar{a} + \bar{b} = \ldots$..	Ans. $4i+3k$
2	If $\bar{a} = 2i + 3j$, $\bar{b} = 3i - j - 2k$, then $\bar{a} - \bar{b}$ *is*	Ans. $-i + 4j + 2k$
3	If $\bar{a} = -i + 3j$, then $\lvert\bar{a}\rvert = \ldots$..	Ans. $\sqrt{10}$
4	$\bar{u} = (1/\sqrt{5})\, i + (2/\sqrt{5})\, j$ then $\lvert\bar{u}\rvert = \ldots$	Ans. 1
5	$\bar{a} = 3i - 4j - 5\sqrt{3}\, k$ then $\lvert\bar{a}\rvert = \ldots$.	Ans. 10
6	If $\bar{a} = -i + 3j$ and $\bar{b} = 2i + 3j$, then $\lvert\bar{a}\rvert + \lvert\bar{b}\rvert$ …	Ans. $\sqrt{10} + \sqrt{13}$
7	$\bar{a} = 2i - 3j$, $\bar{b} = 3j - 4k$ and $\bar{c} = 4k - 2i$ then $\bar{a} + \bar{b} + \bar{c} = \ldots$	Ans. 0
8	If $\bar{a} = 2i + j$ and $\bar{b} = i - 3k$, then $\bar{a} \cdot \bar{b} = \ldots$.	Ans. 2
9	If $\bar{a} = 2i + j + k$ and $\bar{b} = i - j + 3k$, then $\bar{a} \cdot \bar{b} = \ldots$.	Ans .4
10	$\bar{a} = 2i - 2j + k$ and $\bar{b} = i + 3j + 3k$, then $\bar{a} \cdot \bar{b} = \ldots$.	Ans. -1
11	$\bar{a} = 2i + 3j - k$ and $\bar{b} = 4i + 6j - 2k$ then $\bar{a} \times \bar{b} = \ldots$.	Ans. 0
12	$\bar{a} = i + 3j - k$, $\bar{b} = 4i + j - 2k$ then $\bar{a} \times \bar{b} = \ldots$.	Ans $(5i + 2j + 11k)$
13	$\bar{a} = i + j$ and $\bar{b} = j - i$ then angle ($\bar{a}$,^ $\bar{b}$)= …..	Ans. $\pi/2$
14	$\bar{a} = 2i - 3j$, $\bar{b} = i - 3j$ and $\bar{c} = 3i + j$ then $2\bar{a} - (\bar{b} + \bar{c}) = \ldots$.	Ans. $-4j$
15	$\bar{a} = i + 3j$ and $\bar{b} = 5i - j$ then $\lvert\bar{a} + 3\bar{b}\rvert = \ldots$	Ans. 16
16	$\bar{x} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ then $\lvert\bar{x}\rvert = \ldots$.	Ans. 1
17	$\bar{a} \times \bar{a} = \ldots$.	Ans. 0
18	$\bar{a} \cdot \bar{a} = \ldots$.	Ans. $\lvert\bar{a}\rvert^2$
19	$\bar{a} \cdot (\bar{a} \times \bar{b}) = \ldots$..	Ans. 0
20	$(\bar{a} \times \bar{b}) \cdot (\bar{b} \times \bar{a}) = \ldots$.	Ans- $\lvert\bar{a} \times \bar{b}\rvert^2$

Question Set for 03 Marks

1	If $\bar{a}=2i+j-3k$, $\bar{b}=4i+5j+4k$ and $\bar{c}=3i-2j+k$ then find $3\bar{a}+2\bar{b}-3\bar{c}$	$5i+19j-4k$
2	If $\bar{a}=i-2j+4k$, $\bar{b}=-3i+j-4k$ and $\bar{c}=i+2j-4k$ then find $\lvert 5\bar{a}+3\bar{b}+2\bar{c}\rvert$	Ans: $\sqrt{13}$
3	If $\bar{a}=j+k-i$ and $\bar{b}=2i+j-3k$ then find $\lvert 2\bar{a}+3\bar{b}\rvert$	Ans**:** $3\sqrt{10}$
4	If $\bar{a}=3i-j-4k$, $\bar{b}=-2i+4j-3k$ and $\bar{c}=-i+2j-5k$ then find direction cosines of $\bar{a}+2\bar{b}-\bar{c}$.	Ans: $l=0,\ m=\frac{1}{\sqrt{2}},\ n=\frac{-1}{\sqrt{2}}$
5	If $a(1, 0, 0) + b(0, 2, 0) + c\,(0, 0, 3) = (3, 4, 9)$ then find a, b and c.	Ans: $a = 1$, $b = 2$, $c = 3$
6	If $\bar{a}=3i-2j-\sqrt{5}k$ and $\bar{b}=4i+2j+\sqrt{5}k$ then find the projection of $\bar{a}$ on $\bar{b}$.	Ans**:** 3/5
7	If $\bar{a}=i-j+k$, $\bar{b}=2i-j+k$ and $\bar{c}=i+j-2k$ then find $\bar{a}\cdot(\bar{b}+\bar{c})$.	Ans: 2
8	If $\bar{x}=3i-j+2k$ and $\bar{y}=2i+j-k$ then find the vector perpendicular to both $\bar{x}$ and $\bar{y}$.	Ans**:** $\frac{-i+7j+5k}{\sqrt{75}}$
9	If $\bar{a}=10i+2j+3k$, $\bar{b}=i-2j+2k$ and $\bar{c}=3i-2j-2k$ then find $\bar{a}\cdot(\bar{b}\times\bar{c})$	Ans: 4
10	For what value of m, the vectors $2i$-$3j$+$5k$ and mi-$6j$-$8k$ are perpendicular to each other?	m =11
11	For $\bar{x}=(-4,9,6)$, $\bar{y}=(0,7,10)$ and $\bar{z}=(-1,6,6)$ show that $(\bar{x}-\bar{z})\cdot(\bar{y}-\bar{z})=0$.	
12	Show that the angle between the vectors $2i + j + 4k$ and $i + j + k$ is $\cos^{-1}\frac{\sqrt{7}}{3}$	
13	Show that the angle between the vectors $i + j - k$ and $2i - 2j + k$ is $\sin^{-1}\sqrt{\frac{26}{27}}$.	

14	Find a unit vector perpendicular to the vector $\bar{a}=(5,7,-2)$ and $\bar{b}=(3,1,-2)$.	Ans: $\frac{1}{\sqrt{26}}(-3,1,-4)$
15	If $A=i-j-3k$ and $B=j+2i-k$ then prove that $(A\times B)$ is perpendicular to A	

Question Set for <u>04 Marks</u>

1	If $\bar{x}=(1,1,1)$ and $\bar{y}=(2,-1,-1)$ then prove that $\bar{x}$ is perpendicular to $\bar{y}$. Also find an unit vector perpendicular to both $\bar{x}$ and $\bar{y}$.	Ans: $\frac{1}{3\sqrt{2}}(3j-3k)$
2	If $\bar{a}=2i-3j+4k$ and $\bar{b}=i-j+k$ find unit vector perpendicular to $\bar{a}+\bar{b}$ and $\bar{a}-\bar{b}$.	Ans: $\frac{1}{2\sqrt{6}}(-2i-4j-2k)$
3	A body is acted upon the forces $3i - 2j + k$ and $-i - j + 2k$. If the body moves under the forces from the point (2, 2, -3) to (-1, 2, 4), find workdone.	Ans. 15 units
4	A body is acted upon the forces $3i - 2j + 3k$ and $-j + 2k$. If the body moves under the forces from the point (2, 0, -3) to (-1, 2, 2), find workdone.	Ans. 10 units
5	Forces $3i-j+2k$ and $i+3j-k$ are acting on a particle and the particle moves from $2i+3j+k$ to the point $5i+2j+3k$ under these forces. Find the work done by the force.	Ans: 12 units
6	A particle moves form the point $3i - 2j + k$ to the point $i + 3j - 4k$ under the effect of constant forces $i - j + k$, $i + j - 3k$ and $4i + 5j - 6k$. Find the work done.	Ans: 53 units
7	A force F = $2i + j + k$ is acting at the point (-3, 2, 1). Find the magnitude of the moments of force F about the point (2, 1, 2).	Ans: $\sqrt{62}$
8	Find the moment about the point (2, 3, -1) of the force $3i - k$ acting through the point (1,-2, 1). Also find the magnitude of the moment.	Ans: (5, 5, 15) , $5\sqrt{11}$
9	If $x=i+j+k$ and $y=2i-j-k$, then show that x is perpendicular to y. Also find a vector which is perpendicular to both x and y	