Unity in Diversity

Government Polytechnic, Ahmedabad

Science and Humanities Department

Engineering Mathematics

Subject Code: 4320002

Unit -05 Complex Numbers

[Marks – 12]

Course Outcome (CO e):

Solve applied problems using the concept of mean.

Assignment-5

Unit-5 Complex Number

Question set for 01 marks:

1. If $z=2+3i$ then $Re(z)=$ 2

$\rightarrow$ 2

2. If $z=2+i\frac{3}{2}$ then $Im(z)=\frac{3}{2}$

$\rightarrow \frac{3}{2}$

3. Find complex number having $|z|=2$ and $\arg z=\frac{2\pi}{3}$

$\rightarrow -1+i\sqrt{3}$

4. If $P=\frac{1}{\cos\theta-i\sin\theta}$ then $\cos\theta+i\sin\theta=$ P

$\rightarrow \cos\theta+i\sin\theta=P$

5. Modulus of $z=\left(\frac{3}{5}\right)-i\left(\frac{4}{5}\right)$

$\rightarrow |z|=\sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2}=\sqrt{\frac{25}{25}}=\sqrt{1}=\boxed{1}$

6. $(2+3i)\cdot(3-2i)=$ $12+5i$

$6+4i+9i-6i^2=6+5i-6(-1)=6+5i+6=12+5i$

$\rightarrow \boxed{12+5i}$

7. $z=5-3i$ then $\bar{z}=$ $5+3i$

$\rightarrow \bar{z}=5+3i$

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8. If $z_1 = 3 - i, z_2 = 1 + 5i$ then $z_1 - z_2$

$z_1 - z_2 = (3 - i)(1 + 5i)$

$= (3 - 1) + (-i - 5i)$

$= 2 + (+6i)$

$= \boxed{2 - 6i}$

9. $\overline{z_1 - z_2} = \underline{\bar{z}_1 - \bar{z}_2}$

$\rightarrow \bar{z}_1 - \bar{z}_2$

10. If $(x + iy) - (7 + 4i) = 3 - 5i$ then $x = \underline{10}$, $y = \underline{-1}$

Now $x + iy = (3 - 5i) + (7 + 4i)$

$= (3 + 7) + (-5i + 4i)$

$= 10 - i$

$\rightarrow \therefore \boxed{x = 10} \quad \boxed{y = -1}$

11. If $z = \sqrt{3} + i$ then $|z| = \underline{2}$

$|z| = \sqrt{(\sqrt{3})^2 + (1)^2}$

$= \sqrt{3 + 1}$

$= \sqrt{4}$

$\boxed{|z| = 2}$

12 Modulus of $\frac{5 + 12i}{4 + 3i} = \underline{13/5}$

$\rightarrow \frac{13}{5}$

13. Simplify :- $\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^{1/2}$

→ $\frac{1}{2}(1+i\sqrt{3})$

14. If $z = 4-\sqrt{2}i$ then conjugate of z = $\underline{4+\sqrt{2}i}$

→ $\bar{z} = 4+\sqrt{2}i$

conjugate of z = $\boxed{4+\sqrt{2}i}$

15. $i^{4k} = \underline{1}$

→ 1

16. $\sqrt{-4} = \underline{2i}$

$\sqrt{-4} = \sqrt{4}\cdot\sqrt{-1} = \boxed{2i}$

17. Inverse of complex Number $5-4i$ = $\underline{\frac{5+4i}{41}}$

→ $\frac{5+4i}{41}$

[sol^n $z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{5+4i}{(\sqrt{41})^2} = \boxed{\frac{5+4i}{41}}$ Ans

now $\bar{z} = 5+4i$

$|z| = \sqrt{25+16} = \sqrt{41}$

18. $|(3-4i)^2|$

$(3-4i)^2 = 3^2 - 2(3)(4i) + (4i)^2$

$= |7-24i|$

$= \sqrt{7^2+(24)^2}$

$= \sqrt{625}$

$= \boxed{25}$ Ans

19 $\arg(-1) = \underline{\pi}$

$\rightarrow \pi$

(Sol^n $z = -1 = -1 + 0i$ where $-1 < 0$
$\therefore \theta = \arg(z) = \pi$)

20. The conjugate of complex Number $\frac{2-i}{2+i} = \underline{\frac{3+4i}{5}}$

$\rightarrow \boxed{\frac{3+4i}{5}}$

Sol^n $\frac{2-i}{2+i} \times \frac{2-i}{2-i} = \frac{2^2 - 2i - 2i + i^2}{2^2 - i^2}$

$= \frac{4 - 4i - 1}{4 - (-1)} = \frac{3-4i}{5}$

$= \boxed{\frac{3+4i}{5}}$ Ans

(21) $z + \bar{z} = \underline{2Re(z)}$

$\rightarrow$ $2Re(z)$

(22) $z - \bar{z} = \underline{2i\,Im(z)}$

$\rightarrow$ $2i\,Im(z)$

(23) If $|\bar{z}| = 16$ then $|z| = \underline{16}$

$\rightarrow$ $|z| = 16$

(Properties: $|\bar{z}| = |z|$)

(24) $i^9 = \underline{i}$

$\rightarrow$ i

[Sol^n: $i^{4(2)} + i = (1)^2 \cdot i = \boxed{i}$ Ans]

25) $i+i^2+i^3+i^4 = \underline{0}$

$\to 0$

$\underline{\text{Sol}^n}$ $i+(-1)+i^2\cdot i+(i^2)^2$

$= i-1+(-1)\cdot i+(-1)^2$

$= i-\cancel{1}-i+\cancel{1}$

$= i-i = \boxed{0}$

26) $\sqrt{-4} = \underline{2i}$

$\to 2i$

$\underline{\text{Sol}^n}$: $\sqrt{4}\cdot\sqrt{-1}$

$= \boxed{2i}$ Ans

27) $|(3-4i)^2|$

$\to 25$

$\underline{\text{Sol}^n}$ $(a-b)^2 = a^2-2ab+b^2$

$3^2-2(3)(4i)+(4i)^2$

$= 9-24i+16\cdot(-1)$

$= 9-24i-16$

$= -7-24i$

$|(3-4i)^2| = \sqrt{(-7)^2+(24)^2}$

$= \sqrt{49+576}$

$= \sqrt{625}$

$= \boxed{25}$ Ans

28) If $z_1 = 3-2i$ and $z_2 = -2+2i$ then $|z_1+z_2| = \underline{1}$

$\underline{\text{Sol}^n}$: $z_1+z_2 = (3-2i)+(-2+2i)$

$= (3+(-2))+(-2i+2i)$

$= 1+0i$

$= 1$

$|z_1+z_2| = \sqrt{1^2+0^2} = \sqrt{1^2} = \sqrt{1} = \boxed{1}$

29) $\sqrt{-9+0i} = \underline{3i}$

$\to 3i$

$\underline{\text{Sol}^n}$ $\sqrt{-9+0i} = \sqrt{-9}$

$= \sqrt{9}\cdot\sqrt{-1} = 3i$

(30) $\arg(35) = \underline{0}$

→ 0

$z = 35 = 35 + 0i$ where $35 > 0$

$\therefore \theta = \arg(z) = 0$

(31) $(\cos\theta + i\sin\theta)^4 + (\cos\theta + i\sin\theta)^{-4} = \underline{2\cos 4\theta}$

→ $2\cos 4\theta$

[Sol^n: $(\cos 4\theta + i\sin 4\theta) + (\cos 4\theta - i\sin 4\theta)$

$\therefore \boxed{2\cos 4\theta}$ Ans

(32) $(1+i)^{-1} = \underline{\frac{1}{2} - \frac{i}{2}}$

→ $\boxed{\frac{1}{2} - \frac{i}{2}}$

Sol^n here, $a = 1$ and $b = 1$

$$z^{-1} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2} i$$

$$= \frac{1}{1^2+1^2} - \frac{1}{1^2+1^2} i$$

$$= \frac{1}{2} - \frac{1}{2} i$$

$$= \boxed{\frac{1}{2} - \frac{i}{2}}$$ Ans

Question Set for 03 Marks:

1. Find the square root of (1) $3-4\sqrt{10}i$
(2) $7+24i$

Sol^n (1) $3-4\sqrt{10}i$

here $x=3$, $y=-4\sqrt{10}<0$

$$|z| = \sqrt{x^2+y^2} = \sqrt{3^2+(-4\sqrt{10})^2}$$

$$= \sqrt{9+160}$$

$$= \sqrt{169}$$

$$|z| = 13$$

The square root of $3-4\sqrt{10}i$ is

$$= \pm\left(\sqrt{\frac{|z|+x}{2}} - i\sqrt{\frac{|z|-x}{2}}\right)$$

$$= \pm\sqrt{\frac{13+3}{2}} - i\sqrt{\frac{13-3}{2}}$$

$$= \pm\sqrt{\frac{16}{2}} - i\sqrt{\frac{10}{2}}$$

$$= \pm\sqrt{8} - i\sqrt{5}$$

$$= \pm 2\sqrt{2} - i\sqrt{5} = \boxed{\pm 2\sqrt{2}-\sqrt{5}i}$$ Ans

Sol^n (2) $7+24i$

here $x=7$ and $y=24 \geq 0$

$|z| = \sqrt{x^2+y^2} = \sqrt{7^2+24^2} = \sqrt{49+576}$
$= \sqrt{625} = 25$

→ square root of $7+24i$ is

$= \pm\left(\sqrt{\frac{|z|+x}{2}} + i\sqrt{\frac{|z|-x}{2}}\right)$

$= \pm\left(\sqrt{\frac{25+7}{2}} + i\sqrt{\frac{25-7}{2}}\right)$

$= \pm\left(\sqrt{\frac{32}{2}} + i\sqrt{\frac{18}{2}}\right)$

$= \pm\left(\sqrt{16} + i\sqrt{9}\right)$

$= \pm(4+i3)$

$= \boxed{\pm 4+3i}$ Ans

(2) Find the complex conjugate and magnitude of

$\frac{2+3i}{3+2i}$

Sol^n $\frac{2+3i}{3+2i}$

$= \frac{2+3i}{3+2i} \times \frac{3-2i}{3-2i} = \frac{(2+3i)(3-2i)}{3^2-(2i)^2}$

$= \frac{6-4i+9i-6i^2}{9-(4\cdot i^2)}$

$= \frac{6-5i-6(-1)}{9-(4\cdot(-1))} = \frac{6-5i+6}{9+4} = \frac{12-5i}{13}$

conjugate of given complex numbers is $\frac{12+5i}{13}$

magnitude : $\left|\frac{2+3i}{3+2i}\right| = \sqrt{\left(\frac{12}{13}\right)^2+\left(\frac{5}{13}\right)^2}$

$= \sqrt{\frac{144}{169}+\frac{25}{169}} = \sqrt{\frac{169}{169}} = \sqrt{1} = 1$

$= 1.$

$\boxed{\text{conjugate} = \frac{12+5i}{13} \quad \text{and} \quad \text{magnitude} = 1}$ Ans

③ If $z = 3-2i$ then show that $z^2-6z+13=0$

sol^n $z^2-6z+13=0$ (prove)

$z = 3-2i$ (given)

Now $z^2 = (3-2i)^2 = 3^2 - 2(3)(2i) + (2i)^2$

$= 9-12i+(4\cdot i^2)$

$= 9-12i+4(-1)$

$= 9-12i-4$

$= 5-12i$

$6z = 6(3-2i) = 18-12i$

$\therefore z^2 - 6z + 13 = 0$

$L.H.S = \cancel{(18-12i)}$

$= (5-12i) - (18-12i) + 13$

$= 5 - \cancel{12i} - 18 + \cancel{12i} + 13$

$= -13 + 13$

$= 0 = R.H.S$

$\therefore$ prove : $\boxed{z^2 - 6z + 13 = 0}$

(4) Simplify using De Moiver's theorem:

$$\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$\underline{sol^n}$ $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$\therefore (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2) = \cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)$

$= \cos\left(\left(\frac{\pi}{3} + \frac{\pi}{6}\right) + \frac{\pi}{4}\right) + i\sin\left[\left(\frac{\pi}{3} + \frac{\pi}{6}\right) + \frac{\pi}{4}\right]$

$= \left[\cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right)\right] + i\sin\left[\frac{\pi}{2} + \frac{\pi}{4}\right]$

$= -\sin\frac{\pi}{4} + i\cos\frac{\pi}{4}$

$= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$

$= -\frac{1}{\sqrt{2}} (1-i)$

$= \boxed{\frac{1}{\sqrt{2}} (-1+i)}$ Ans

Q-5 Simplify using De-moivre's theorem

$$\frac{(\cos 2\theta + i\sin 2\theta)^4 \cdot (\cos\theta - i\sin\theta)^2}{(\cos\theta + i\sin\theta)^3 \cdot (\cos 3\theta - i\sin 3\theta)^4}$$

Sol^n $\frac{(\cos 2\theta + i\sin 2\theta)^4 \cdot (\cos\theta - i\sin\theta)^2}{(\cos\theta + i\sin\theta)^3 \cdot (\cos 3\theta - i\sin 3\theta)^4}$

$= \frac{[(\cos\theta + i\sin\theta)^2]^4 \cdot (\cos\theta + i\sin\theta]^{-2}}{[\cos\theta + i\sin\theta]^3 \cdot [(\cos\theta + i\sin\theta)^{-3}]^4}$

$= \frac{(\cos\theta + i\sin\theta)^8 \cdot (\cos\theta + i\sin\theta)^{-2}}{(\cos\theta + i\sin\theta)^3 \cdot (\cos\theta + i\sin\theta)^{-12}}$

$= \frac{(\cos\theta + i\sin\theta)^6}{(\cos\theta + i\sin\theta)^{-9}}$

$= (\cos\theta + i\sin\theta)^{6-(-9)}$

$= (\cos\theta + i\sin\theta)^{15}$

$= \boxed{\cos 15\theta + i\sin 15\theta}$ Ans

6 If $z = x+iy$ then find locus of $\left|\frac{z-2i}{z+2i}\right| = \sqrt{2}$

Solⁿ $\left|\frac{z-2i}{z+2i}\right| = \sqrt{2}$

$\therefore \left|\frac{x+iy-2i}{x+iy+2i}\right| = \sqrt{2}$

$\therefore \left|\frac{x+i(y-2)}{x+i(y+2)}\right| = \sqrt{2}$

$\therefore \frac{\sqrt{x^2+i^2(y-2)^2}}{\sqrt{x^2+i^2(y+2)^2}} = \sqrt{2}$

$=$ Sq. Both side $\quad (\because i = \sqrt{-1},\ i = 1)$

$\Rightarrow x^2 + (y-2)^2 = 2\cdot(x^2+(y+2)^2$

$\therefore x^2 + y^2 - 2(y)(2) + 4 = 2(x^2+y^2+2(y)(2)+4)$

$\therefore x^2+y^2-4y+4 = 2(x^2+y^2+4y+4)$

$\therefore x^2+y^2-4y+4 = 2x^2+2y^2+8y+8$

$\therefore x^2-2x^2+y^2-2y^2-4y-8y+4-8=0$

$\therefore -x^2-y^2-12y-4=0$

$$\boxed{x^2+y^2+12y+4=0} \text{ Ans}$$

(7) If $\frac{(1+i)^2}{2-i} = x+iy$ then find the value of $x+y$.

Sol^n $x+iy = \frac{(1+i)^2}{2-i} = \frac{1^2+2(1)(i)+i^2}{2-i} = \frac{1+2i+i^2}{2-i} = \frac{1+2i-1}{2-i}$

$$x+iy = \frac{2i}{2-i} \times \frac{2+i}{2+i}$$

$$= \frac{2i(2+i)}{2^2-i^2}$$

$$= \frac{4i+2i^2}{4-i^2}$$

$$= \frac{4i-2}{4+1} = \frac{4i-2}{5} = \frac{-2+4i}{5}$$

$$x+iy = \frac{-2+4i}{5}$$

$$= \frac{-2}{5} + \frac{4}{5}i$$

then $x = \frac{-2}{5}$ and $y = \frac{4}{5}$

$$\therefore x+y = \frac{-2}{5} + \frac{4}{5}$$

$$= \frac{-2+4}{5} = \boxed{\frac{2}{5}} \text{ Ans}$$

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(6) Prove that $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1} \cdot \cos \frac{n\pi}{6}, n>0$

Sol^n $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1} \cdot \cos \frac{n\pi}{6}$

L.H.S $: (\sqrt{3}+i)^n + (\sqrt{3}-i)^n$

$$= \left(2 \cdot \frac{\sqrt{3}}{2} + 2 \cdot \frac{i}{2}\right)^n + \left(2 \cdot \frac{\sqrt{3}}{2} - 2 \cdot \frac{i}{2}\right)^n$$

$$= 2^n \left(\frac{\sqrt{3}}{2} \cdot \frac{i}{2}\right)^n + 2^n \left[\frac{\sqrt{3}}{2} - \frac{1}{2} i\right]^n$$

$$= 2^n \left[\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)^n + \left(\frac{\sqrt{3}}{2} - \frac{1}{2} i\right)^n\right]$$

$$= 2^n \left[\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)^n + \left(\cos \frac{\pi}{6} - i \sin \frac{\pi}{6}\right)^n\right]$$

$$= 2^n \left[\cos \frac{n\pi}{6} + \cancel{i \sin \frac{n\pi}{6}}\right) + \cos \frac{n\pi}{6} - \cancel{i \sin \frac{n\pi}{6}}\Big)$$

$$= 2^n \left(2 \cos \frac{n\pi}{6}\right)$$

$$= \boxed{2^{n+1} \cdot \cos \frac{n\pi}{6}} \quad \text{Ans}$$

(1) Simplify $\frac{(1-i)\cdot(2-i)\cdot(3-i)}{1+i}$ into $a+ib$ form

Sol: $\frac{(1-i)(2-i)(3-i)}{1+i}$

$= \frac{(2-i)(3-i)(1-i)}{(1+i)} \times \frac{(1-i)}{1-i}$ (multiply by conjugate)

$= \frac{[(2-i)(3-i)](1-i)^2}{1^2-(i)^2}$

$= \frac{[6-2i-3i+i^2][1^2-2(1)(i)+(i^2)]}{1-(-1)}$

$= \frac{(6-5i-1)(1-2i-1)}{2}$

$= \frac{(5-5i)(-2i)}{2}$

$= (5-5i)(-i)$

$= -5i+5i^2$

$= -5i-5$

$= \boxed{-5-5i}$ Ans

10. Find $x, y \in R$ from the equation

$(7x+iy)(2-i) = 5(2-3i)+2(1+2i)$

$\therefore (7x+iy)(2-i) = 5(2-3i)+2(1+2i)$

$\therefore 14x - 7xi + 2yi - yi^2 = 10 - 15i + 2 + 4i$

$\therefore 14x - y(-1) - 7xi + 2yi = 12 - 11i$

$\therefore (14x+y) + i(2y-7x) = 12 - 11i$

Comparing real and imaginary parts of both the sides.

$14x + y = 12$ ① and $2y - 7x = -11$ ②

$\therefore y = 12 - 14x$

put the value of y in eqn ②

$2y - 7x = -11$

$2(12-14x) - 7x = -11$

$24 - 28x - 7x = -11$

$24 - 35x = -11$

$x = \frac{-11-24}{-35}$

$\boxed{x = 1}$

put the $x=1$ in eqn ①

$y = 12 - 14(1)$

$= 12 - 14$

$= \boxed{-2}$

$\therefore \boxed{x = 1}$ and $\boxed{y = -2}$ Ans

(11) If $z = \cos\theta + i\sin\theta$, then show that

$z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i\sin n\theta$

Solⁿ $L.H.S = z^n + \frac{1}{z^n}$

$= z^n + z^{-n}$

$= (\cos\theta + i\sin\theta)^n + (\cos\theta + i\sin\theta)^{-n}$

$= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$

$= 2\cos n\theta$

$= R.H.S$

$L.H.S = z^n - \frac{1}{z^n}$

$= z^n - z^{-n}$

$= (\cos\theta + i\sin\theta)^n - (\cos\theta + i\sin\theta)^{-n}$

$= \cos n\theta + i\sin n\theta - (\cos n\theta - i\sin n\theta)$

$= \cos n\theta + i\sin n\theta - \cos n\theta + i\sin n\theta$

$= 2i\sin n\theta$

$= R.H.S$

(12) Find the modulus and principal argument of $z = \frac{1+i}{1-i}$ and express z into polar form.

Solⁿ $\frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{1+2i+i^2}{1-i^2} = \frac{1+2i-1}{1+1} = \frac{2i}{2}$

$\therefore z = i = 0 + i$

$\therefore x = 0$ and $y = 1 > 0$

$\therefore r = |z| = \boxed{1}$ and the principal argument of z is

$\theta = \arg(z) = \boxed{\pi/2}$

$\therefore$ Polar form of $z = r(\cos\theta + i\sin\theta)$

$= 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$= \boxed{\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}}$ Ans

Q-13 Find the inverse of complex number $\frac{2+3i}{4-3i}$

Sol^n

$z^{-1} = \frac{4-3i}{2+3i} = \frac{4-3i}{2+3i} \times \frac{2-3i}{2-3i}$

$= \frac{(4-3i)(2-3i)}{2^2 - (3i)^2}$

$= \frac{8 - 12i - 6i + 9i^2}{4 - 9(i^2)}$

$= \frac{8 - 18i + 9(-1)}{4 - 9(-1)}$

$$z^{-1} = \frac{8 - 9 - 18i}{4+9}$$

$$= \frac{-1 - 18i}{13}$$

$$z^{-1} = \boxed{\frac{-1}{13} - \frac{18}{13} i}$$ Ans

Q-14 If $z = -3 + \sqrt{2}i$ then find the value of $z^4 + 5z^3 + 8z^2 + 7z + 4$

Sol^n

$z = -3 + \sqrt{2}i$

$\therefore z + 3 = \sqrt{2}i$

$\therefore (z+3)^2 = (\sqrt{2}i)^2$ (square of both the sides)

$z^2 + 2(z)(3) + 9 = 2i^2$

$\therefore z^2 + 6z + 9 = -2$

$\therefore z^2 + 6z + 9 + 2 = 0$

$\therefore z^2 + 6z + 11 = 0$ --- (1)

Now

$$\begin{array}{r|l} & z^2 - z + 3 \\ \hline z^2 + 6z + 11 & z^4 + 5z^3 + 8z^2 + 7z + 4 \\ & \cancel{z^4} + 6z^3 + 11z^2 \\ & - \quad - \quad - \\ \hline & -\cancel{z^3} - 3z^2 + 7z + 4 \\ & -\cancel{z^3} - 6z^2 - 11z \\ & (+) \quad (+) \quad (+) \\ \hline & 3z^2 + 18z + 4 \\ & 3z^2 + 18z + 33 \\ & (-) \quad (-) \quad (-) \\ \hline & -29 \end{array}$$

$\therefore \quad z^4 + 5z^3 + 8z^2 + 7z + 4$

$= (z^2 + 6z + 11)(z^2 - z + 3) - 29$

$= 0\,(z^2 - z + 3) - 29$ (from eqn (1))

$= 0 - 29$

$= \boxed{-29}$ Ans

Q-15 For $z = 1 + i$ find $|\bar{z}|$ and $\arg(z)$

Soln $z = 1 + i$

comparing with $x + iy$

$x = 1, y = 1$

$\bar{z} = 1 - i$ Now $x = 1, y = -1$

so $|\bar{z}| = \sqrt{x^2 + y^2}$

$= \sqrt{(1)^2 + (-1)^2}$

$= \sqrt{1 + 1}$

$\boxed{|\bar{z}| = \sqrt{2}}$ Ans

→ Argument or Amplitude

angle $= \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{+1}{1}\right)$

$= \tan^{-1}(1)$

$\boxed{\theta = \arg(z) = \pi/4}$ Ans

Q-16 Simplify $\left(\frac{\cos 3\theta + i\sin 3\theta}{\cos\theta - i\sin\theta}\right)^2$

Sol^n

$$\left(\frac{\cos 3\theta + i\sin 3\theta}{\cos\theta - i\sin\theta}\right)^2$$

$$= \frac{\left[(\cos\theta + i\sin\theta)^3\right]^2}{\left[(\cos\theta - i\sin\theta)^{-1}\right]^2}$$

$$= \left[\frac{(\cos\theta + i\sin\theta)^3}{(\cos\theta + i\sin\theta)^{-1}}\right]^2$$

$$= \left((\cos\theta + i\sin\theta)^{3-(-1)}\right)^2$$

$$= \left[(\cos\theta + i\sin\theta)^4\right]^2$$

$$= (\cos\theta + i\sin\theta)^8$$

$$= \boxed{\cos 8\theta + i\sin 8\theta} \quad \underline{\text{Ans}}$$

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* Question set for 04 marks:

1. Find square root of $z = -2 + 2\sqrt{3}$

Soln $z = -2 + 2\sqrt{3}$

$\therefore x = -2, y = 2\sqrt{3}$

$|z| = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{4 + (4 \times 3)} = \sqrt{4+12} = \sqrt{16}$
$= 4$

As $y = 2\sqrt{3} > 0$ square root of $z = -2 + 2\sqrt{3}i$

are $\pm \sqrt{\frac{|z| + x}{2}} + i \sqrt{\frac{|z| - x}{2}}$

$\pm \sqrt{\frac{4 + (-2)}{2}} + i \sqrt{\frac{4 - (-2)}{2}}$

$\pm \sqrt{\frac{2}{2}} + i \sqrt{\frac{6}{2}}$

$\pm \sqrt{1} + i\sqrt{3}$

$\boxed{\pm (1 + i\sqrt{3})}$ Ans

(7) Find the modulus of amplitude of $z = \dfrac{(3i-2)(1+2i)}{i(2+i)}$

Sol^n $z = \dfrac{(3i-2)(1+2i)}{i(2+i)}$

$= \dfrac{(-2+3i)(1+2i)}{2i+i^2}$

$= \dfrac{-i-8}{-1+2i}$

$= \dfrac{-8-i}{-1+2i} \times \dfrac{-1-2i}{-1-2i}$

$= \dfrac{8+16i+i+2i^2}{(-1)^2-(2i)^2} = \dfrac{6+17i}{5} = \dfrac{6}{5} + \dfrac{17}{5}i$

Now $|z| = \sqrt{x^2+y^2} \quad (\because x = \frac{6}{5}, y = \frac{17}{5})$

$= \sqrt{\left(\frac{6}{5}\right)^2 + \left(\frac{17}{5}\right)^2}$

$= \sqrt{\frac{36}{25} + \frac{289}{25}} = \sqrt{\frac{325}{25}} = \sqrt{13}$

modulus of $z = \boxed{|z| = \sqrt{13}}$ Ans

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Amplitude of z

$\tan\theta = \left|\frac{y}{x}\right| = \left|\frac{17/5}{6/5}\right| = \frac{17}{6}$

Amplitude of z

$\boxed{\theta = \tan^{-1}\left(\frac{17}{6}\right)}$ Ans

Q-4 Prove that :- $\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4 = \cos 8\theta + i\sin 8\theta$

L.H.S $\left(\frac{\cos\theta + i\sin\theta}{\cos(\pi/2 - \theta) + i\sin(\pi/2 - \theta)}\right)^4$

$= \left(\frac{e^{i\theta}}{e^{i(\pi/2-\theta)}}\right)^4$ $(\because \cos\theta + i\sin\theta = e^{i\theta})$

$= \left(e^{i\theta - i\pi/2 + i\theta}\right)^4$

$= e^{i(2\theta - \pi/2)4}$

$= e^{i(8\theta - 2\pi)}$

$= \cos(8\theta - 2\pi) + i\sin(8\theta - 2\pi)$

$= \boxed{\cos 8\theta + i\sin 8\theta}$ Ans

Q.5. Simplify :- $\frac{(\cos 3\alpha + i\sin 3\alpha)^5 (\cos 3\alpha + i\sin 3\alpha)^2}{(\cos 4\alpha + i\sin 4\alpha)^3}$

Soln $\frac{(\cos 3\alpha + i\sin 3\alpha)^5 \cdot (\cos 3\alpha + i\sin 3\alpha)^2}{(\cos 4\alpha + i\sin 4\alpha)^3}$

$= \frac{[(\cos\alpha + i\sin\alpha)^3]^5 \cdot [(\cos\alpha + i\sin\alpha)^3]^2}{[(\cos\alpha + i\sin\alpha)^4]^3}$

$= \frac{(\cos\alpha + i\sin\alpha)^{15} \cdot (\cos\alpha + i\sin\alpha)^6}{(\cos\alpha + i\sin\alpha)^{12}}$

$= \frac{(\cos\alpha + i\sin\alpha)^{15+6}}{(\cos\alpha + i\sin\alpha)^{12}}$

$= \frac{(\cos\alpha + i\sin\alpha)^{21}}{(\cos\alpha + i\sin\alpha)^{12}}$

$= (\cos\alpha + i\sin\alpha)^{21-12}$

$= (\cos\alpha + i\sin\alpha)^9$

$= \boxed{\cos 9\alpha + i\sin 9\alpha}$ Ans

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Dt.: / / Pg.no.:

Q-6 Prove that:

$$\frac{(\cos 11\theta + i\sin 11\theta)^2 (\cos\theta - i\sin\theta)^3}{(\cos 2\theta + i\sin 2\theta)^{11} \cdot (\cos 3\theta + i\sin 3\theta)^1} = \cos 6\theta - i\sin 6\theta$$

Sol^n L.H.S

$$= \frac{[(\cos\theta + i\sin\theta)^{11}]^2 \cdot (\cos\theta + i\sin\theta)^{-3}}{[(\cos\theta + i\sin\theta)^2]^{11} \cdot (\cos\theta + i\sin\theta)^{3\cdot 1}}$$

$$= \frac{(\cos\theta + i\sin\theta)^{22} \cdot (\cos\theta + i\sin\theta)^{-3}}{(\cos\theta + i\sin\theta)^{22} \cdot (\cos\theta + i\sin\theta)^{3}}$$

$$= \frac{(\cos\theta + i\sin\theta)^{22+(-3)}}{(\cos\theta + i\sin\theta)^{22+3}}$$

$$= \frac{(\cos\theta + i\sin\theta)^{19}}{(\cos\theta + i\sin\theta)^{25}}$$

$$= (\cos\theta + i\sin\theta)^{19-25}$$

$$= (\cos\theta + i\sin\theta)^{-6}$$

$$= \boxed{\cos 6\theta - i\sin 6\theta} \quad \text{R.H.S}$$

Q.7 If $z_1 = 2-3i$ and $z_2 = 3-2i$ then find z_1+z_2, z_1-z_2, $z_1 \times z_2$ and $\frac{z_1}{z_2}$

Solⁿ $z_1 = 2-3i$ and $z_2 = 3-2i$

$$z_1+z_2 = (2-3i)+(3-2i)$$
$$= (2+3)+(-3-2)i$$
$$= 5+(-5)i$$
$$\boxed{z_1+z_2 = 5-5i}$$

$$z_1-z_2 = (2-3i)-(3-2i)$$
$$= (2-3)+i(-3-(-2))$$
$$= -1+i(-3+2)$$
$$= -1+i(-1)$$
$$\boxed{z_1-z_2 = -1-i}$$

$$z_1 \times z_2 = (2-3i)(3-2i)$$
$$= 6-4i-9i+6i^2$$
$$= 6-13i+6i^2$$
$$= 6-13i+6(-1) \quad (\because i^2=-1)$$
$$= \cancel{6}-13i-\cancel{6}$$
$$= -13i$$
$$\boxed{z_1 \times z_2 = 0-13i}$$

$$\frac{z_1}{z_2} = \frac{2-3i}{3-2i} \times \frac{3+2i}{3+2i}$$

$$= \frac{(2-3i)(3+2i)}{3^2-(2i)^2}$$

$$= \frac{6+4i-9i-6i^2}{9-(4\times i^2)}$$

$$= \frac{6-5i-6i^2}{9-(4\times(-1))}$$

$$= \frac{6-5i-6(-1)}{9-(-4)}$$

$$= \frac{6-5i+6}{9+4}$$

$$= \frac{12-5i}{13}$$

$$= \boxed{\frac{12}{13} - \frac{5}{13}i}$$

Q-8 If $z_1 = 1+i$ and $z_2 = 2-i$ then show that

$$|z_1+z_2|^2 + |z_1-z_2|^2 = 2\left(|z_1|^2 + |z_2|^2\right)$$

Sol^n $L.H.S = |z_1+z_2|^2 + |z_1-z_2|^2$

first find the value of z_1+z_2 and z_1-z_2

$\therefore\ z_1+z_2 = (1+i)+(2-i) = 3+0i$

$\therefore\ |z_1+z_2| = \sqrt{3^2+0^2} = \sqrt{9} = 3$

$\therefore\ |z_1+z_2|^2 = 3^2 = 9$

now $z_1 - z_2 = (1+i) - (2-i)$

$= (1-2) + (1-(-1))i$

$= -1 + 2i$

$|z_1 - z_2| = \sqrt{(-1)^2 + (2)^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$

$|z_1 - z_2|^2 = (\sqrt{5})^2 = 5$

Then

$|z_1 + z_2|^2 + |z_1 - z_2|^2$

$= 9 + 5$

$= \boxed{14.}$

R.H.S $= 2(|z_1|^2 + |z_2|^2)$

$|z_1| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$|z_1|^2 = (\sqrt{2})^2 = 2$

$|z_2| = \sqrt{2^2 + (-1)^2}$

$= \sqrt{4+1} = \sqrt{5}$

$|z_2|^2 = (\sqrt{5})^2 = 5$

$= 2(2+5)$

$= 2(7)$

$= \boxed{14.}$

$\therefore$ Prove $\boxed{L.H.S = R.H.S}$

Q.10 If $z = \dfrac{3}{1+\sqrt{2}i}$ then prove that $z^2 - 2z + 3 = 0$. Find the value of $z^3 + z^2 - 3z + 10$ using it.

S.10

$$z = \frac{3}{1+\sqrt{2}i}$$

$$= \frac{3}{1+\sqrt{2}i} \times \frac{1-\sqrt{2}i}{1-\sqrt{2}i}$$

$$= \frac{3(1-\sqrt{2}i)}{1^2 - (\sqrt{2}i)^2}$$

$$= \frac{3(1-\sqrt{2}i)}{1-(-2)}$$

$$= \frac{\cancel{3}(1-\sqrt{2}i)}{\cancel{3}}$$

$$z = 1-\sqrt{2}i$$

$z = 1-\sqrt{2}i$

$\therefore z - 1 = -\sqrt{2}i$

squaring both the sides

$(z-1)^2 = (-\sqrt{2}i)^2$

$\therefore z^2 - 2(z)(1) + 1^2 = 2i^2$

$\therefore z^2 - 2z + 1 = 2(-1)$ $\quad (\because i^2 = -1)$

$\therefore z^2 - 2z + 1 + 2 = 0$

$\therefore z^2 - 2z + 3 = 0$

Now

$$
\begin{array}{r|l}
 & z + 3 \\
\hline
z^2 - 2z + 3 & \cancel{z^3} + z^2 - 3z + 10 \\
 & \cancel{z^3} - 2z^2 + 3z \\
 & - \quad + \quad - \\
\hline
 & 3\cancel{z^2} - 6\cancel{z} + 10 \\
 & 3\cancel{z^2} - 6\cancel{z} + 9 \\
 & - \quad + \quad - \\
\hline
 & 1
\end{array}
$$

$\therefore z^3 + z^2 - 3z + 10 = (z^2 - 2z + 3)(z+3) + 1$

$= 0(z+3) + 1 = 0 + 1 = \boxed{1}$ Ans